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LIMIT AUTOMORPHISMS OF THE C^*-ALGEBRAS GENERATED BY ISOMETRIC REPRESENTATIONS FOR SEMIGROUPS OF RATIONALS

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Abstract: We consider inductive sequences of Toeplitz algebras whose connecting homomorphisms are defined by collections of primes. The inductive limits of these sequences are C^*-algebras generated by representations for semigroups of rationals. We study the limit endomorphisms of these C^*-algebras induced by morphisms between copies of the same inductive sequences of Toeplitz algebras. We establish necessary and sufficient conditions for these endomorphisms to be automorphisms of the algebras.

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Introduction

It is well known that the properties of objects and morphisms in the categories of Banach algebras have corresponding analogs in algebraic and topological categories and vice versa (see, for example, [1, 2]).

This article was motivated by several sources. On the one hand, these sources include the articles [3–11] on Toeplitz algebras which are also called semigroup C^*-algebras. On the other hand, they also comprise the papers [12–18] which are devoted to mappings of topological groups. The article [19] contains an application of the properties of mappings of P-adic solenoids to crossed products of C^*-algebras.

In [3, 4], Coburn studied the Toeplitz algebra for the additive semigroup of nonnegative integers. In [5], Douglas considered the case of subsemigroups of the additive group of the reals. In [6, 8], Murphy studied the general case of ordered groups and, in particular, proved that the correspondence between ordered groups and Toeplitz algebras is a continuous functor. These authors showed that isometric representations of semigroups have the universality property (see § 1). This property is involved in our constructions of $*$-homomorphisms between the C^*-algebras generated by nonunitary isometries.

This article is devoted to the limit endomorphisms of Toeplitz algebras that are generated by isometric representations of additive semigroups of nonnegative rationals. We consider these semigroup C^*-algebras as the inductive limits of inductive sequences of Toeplitz algebras with connecting homomorphisms defined by sequences of primes. By a limit endomorphism we mean a $*$-homomorphism induced by a morphism between two copies of the same inductive sequence of algebras. We prove the properties of limit endomorphisms that can be regarded as the corresponding operator-algebraic analogs of the properties of mappings of P-adic solenoids (see, for example, [12, 13, 15, 16, 20–22]) and group homomorphisms of rationals. Here we do not use functorial constructions such as the above-mentioned continuous functor and the Pontryagin duality (see Remark 4 in § 3 concerning the use of functors for the proofs).

The article consists of an Introduction and three sections. In § 1, we introduce the notations and recall the definitions and results of use in the sequel. Then, in § 2, we consider the inductive sequences of Toeplitz algebras defined by sequences of primes. § 3 is devoted to limit endomorphisms and contains the main results of the article. We establish the necessary and sufficient conditions for these endomorphisms to be $*$-automorphisms. The conditions are formulated in number-theoretic, algebraic, and functional terms.

§ 1. Notations and Prerequisites

Throughout the article, $P = (p_1, p_2, \dots)$ stands for an arbitrary sequence of primes (1 is not listed

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